

Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Monday 24 May 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 11 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Write

$$\sqrt{(75)} - \sqrt{(27)}$$

in the form $k\sqrt{x}$, where k and x are integers.

(2)

2. Find

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5) \, dx,$$

giving each term in its simplest form.

(4)

3. Find the set of values of x for which

(a) $3(x - 2) < 8 - 2x,$

(2)

(b) $(2x - 7)(1 + x) < 0,$

(3)

(c) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0.$

(1)

4. (a) Show that $x^2 + 6x + 11$ can be written as

$$(x + p)^2 + q,$$

where p and q are integers to be found.

(2)

(b) Sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

(c) Find the value of the discriminant of $x^2 + 6x + 11.$

(2)

5. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)}, \quad n \geq 1,$$
$$a_1 = 2.$$

- (a) Find a_2 and a_3 , leaving your answers in surd form.

(2)

- (b) Show that $a_5 = 4$.

(2)

6.

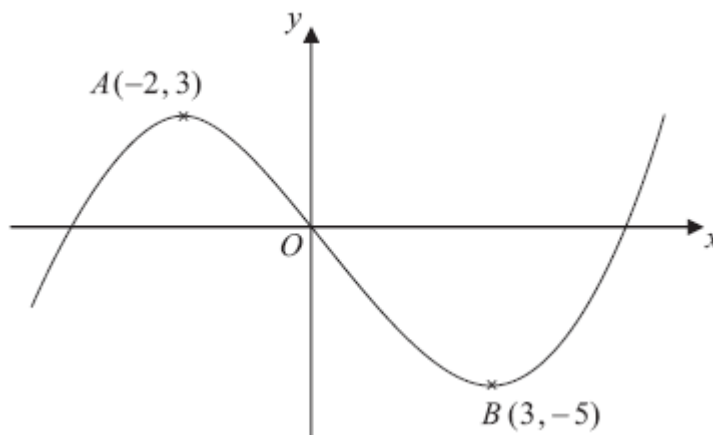


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve has a maximum point A at $(-2, 3)$ and a minimum point B at $(3, -5)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 3)$,

(3)

(b) $y = 2f(x)$.

(3)

On each diagram show clearly the coordinates of the maximum and minimum points.

The graph of $y = f(x) + a$ has a minimum at $(3, 0)$, where a is a constant.

- (c) Write down the value of a .

(1)

7. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0,$$

find $\frac{dy}{dx}$.

(6)

8. (a) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(3)

(b) Find the length of AB , leaving your answer in surd form.

(2)

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of t .

(1)

(d) Find the area of triangle ABC .

(2)

9. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays $\pounds a$ for their first day, $\pounds(a + d)$ for their second day, $\pounds(a + 2d)$ for their third day, and so on, thus increasing the daily payment by $\pounds d$ for each extra day they work.

A picker who works for all 30 days will earn $\pounds 40.75$ on the final day.

(a) Use this information to form an equation in a and d .

(2)

A picker who works for all 30 days will earn a total of $\pounds 1005$.

(b) Show that $15(a + 40.75) = 1005$.

(2)

(c) Hence find the value of a and the value of d .

(4)

10. (a) On the axes below sketch the graphs of

(i) $y = x(4 - x)$,

(ii) $y = x^2(7 - x)$,

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(5)

(b) Show that the x -coordinates of the points of intersection of

$$y = x(4 - x) \quad \text{and} \quad y = x^2(7 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$.

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A , leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where p , q , r and s are integers.

(7)

11. The curve C has equation $y = f(x)$, $x > 0$, where

$$\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2.$$

Given that the point $P(4, 5)$ lies on C , find

(a) $f(x)$,

(5)

(b) an equation of the tangent to C at the point P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(4)

TOTAL FOR PAPER: 75 MARKS

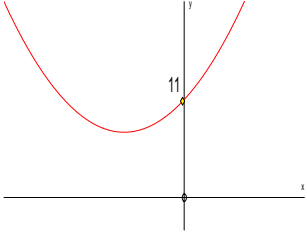
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June 2010
Core Mathematics C1 6663
Mark Scheme

Question Number	Scheme	Marks
1.	$(\sqrt{75} - \sqrt{27}) = 5\sqrt{3} - 3\sqrt{3}$ $= 2\sqrt{3}$	M1 A1 <p style="text-align: right;">2</p>
	<u>Notes</u>	
	M1 for $5\sqrt{3}$ from $\sqrt{75}$ or $3\sqrt{3}$ from $\sqrt{27}$ seen anywhere A1 for $2\sqrt{3}$; allow $\sqrt{12}$ or $k = 2, x = 3$ allow $k = 1, x = 12$ <u>Some Common errors</u> $\sqrt{75} - \sqrt{27} = \sqrt{48}$ leading to $4\sqrt{3}$ is M0A0 $25\sqrt{3} - 9\sqrt{3} = 16\sqrt{3}$ is M0A0	

Question Number	Scheme	Marks
2.	$\frac{8x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + c$ $= 2x^4 + 4x^{\frac{3}{2}} - 5x + c$	<p>M1 A1</p> <p>A1 A1</p> <p style="text-align: right;">4</p>
Notes		
<p>M1 for some attempt to integrate a term in x: $x^n \rightarrow x^{n+1}$</p> <p>1st A1 for correct, possibly un-simplified x^4 or $x^{\frac{3}{2}}$ term. e.g. $\frac{8x^4}{4}$ or $\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$</p> <p>2nd A1 for <u>both</u> $2x^4$ and $4x^{\frac{3}{2}}$ terms correct and simplified on the same line N.B. some candidates write $4\sqrt{x^3}$ or $4x^{1\frac{1}{2}}$ which are, of course, fine for A1</p> <p>3rd A1 for $-5x + c$. Accept $-5x^1 + c$. The $+c$ must appear on the same line as the $-5x$ N.B. We do not need to see one line with a fully correct integral</p> <p>Ignore ISW (ignore incorrect subsequent working) if a correct answer is followed by an incorrect version.</p> <p>Condone poor use of notation e.g. $\int 2x^4 + 4x^{\frac{3}{2}} - 5x + c$ will score full marks.</p>		

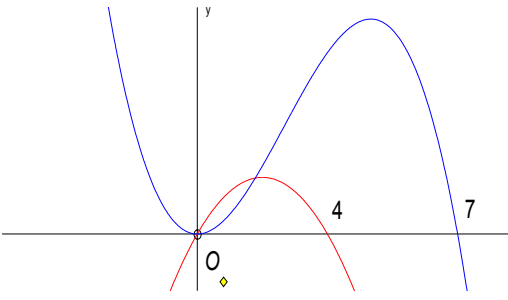
Question Number	Scheme	Marks
3.		
(a)	$3x - 6 < 8 - 2x \rightarrow 5x < 14$ (Accept $5x - 14 < 0$ (o.e.)) $x < 2.8$ or $\frac{14}{5}$ or $2\frac{4}{5}$ (condone \leq)	M1 A1 (2)
(b)	Critical values are $x = \frac{7}{2}$ and -1 Choosing "inside" $-1 < x < \frac{7}{2}$	B1 M1 A1 (3)
(c)	$-1 < x < 2.8$	B1ft (1)
Accept any exact equivalents to -1, 2.8, 3.5		6
Notes		
(a)	M1 for attempt to rearrange to $kx < m$ (o.e.) Either $k = 5$ or $m = 14$ should be correct Allow $5x = 14$ or even $5x > 14$	
(b)	B1 for both correct critical values. (May be implied by a correct inequality) M1 ft their values and choose the "inside" region A1 for fully correct inequality (Must be in part (b): do not give marks if only seen in (c)) Condone seeing $x < -1$ in working provided $-1 < x$ is in the final answer. e.g. $x > -1$, $x < \frac{7}{2}$ <u>or</u> $x > -1$ "or" $x < \frac{7}{2}$ <u>or</u> $x > -1$ "blank space" $x < \frac{7}{2}$ score M1A0 BUT allow $x > -1$ and $x < \frac{7}{2}$ to score M1A1 (the "and" must be seen) Also $(-1, \frac{7}{2})$ will score M1A1 NB $x < -1, x < \frac{7}{2}$ is of course M0A0 and a number line even with "open" ends is M0A0 Allow 3.5 instead of $\frac{7}{2}$	
(c)	B1ft for $-1 < x < 2.8$ (ignoring their previous answers) <u>or</u> ft their answers to part (a) and part (b) provided both answers were regions and not single values. Allow use of "and" between inequalities as in part (b) If their set is empty allow a suitable description in words or the symbol \emptyset . <u>Common error:</u> If (a) is correct and in (b) they simply leave their answer as $x < -1$, $x < 3.5$ then in (c) $x < -1$ would get B1ft as this is a correct follow through of these 3 inequalities. Penalise use of \leq only on the A1 in part (b). [i.e. condone in part (a)]	

Question Number	Scheme	Marks
4. (a)	$(x+3)^2 + 2$ <p style="text-align: center;">or $p = 3$ or $\frac{6}{2}$ $q = 2$</p>	B1 B1 (2)
(b)	 <p style="text-align: center;">U shape with min in 2nd quad (Must be above x-axis and not on y=axis)</p> <p style="text-align: center;">U shape crossing y-axis at (0, 11) only (Condone (11,0) marked on y-axis)</p>	B1 B1 (2)
(c)	$b^2 - 4ac = 6^2 - 4 \times 11$ $= \underline{-8}$	M1 A1 (2) 6
Notes		
(a)	Ignore an “= 0” so $(x+3)^2 + 2 = 0$ can score both marks	
(b)	<p>The U shape can be interpreted fairly generously. Penalise an obvious V on 1st B1 only. The U needn't have equal “arms” as long as there is a clear min that “holds water”</p> <p>1st B1 for U shape with minimum in 2nd quad. Curve need not cross the y-axis but minimum should NOT touch x-axis and should be left of (not on) y-axis</p> <p>2nd B1 for U shaped curve crossing at (0, 11). Just 11 marked on y-axis is fine. The point must be marked on the sketch (do not allow from a table of values) Condone stopping at (0, 11)</p>	
(c)	<p>M1 for some correct substitution into $b^2 - 4ac$. This may be as part of the quadratic formula but must be in part (c) and must be only numbers (no x terms present). Substitution into $b^2 < 4ac$ or $b^2 = 4ac$ or $b^2 > 4ac$ is M0</p> <p>A1 for - 8 only. If they write $- 8 < 0$ treat the < 0 as ISW and award A1 If they write $- 8 \geq 0$ then score A0 A substitution in the quadratic formula leading to - 8 inside the square root is A0. So substituting into $b^2 - 4ac < 0$ leading to $- 8 < 0$ can score M1A1.</p> <p>Only award marks for use of the discriminant in part (c)</p>	

Question Number	Scheme	Marks
7.	$\frac{3x^2 + 2}{x} = 3x + 2x^{-1}$ $(y' =) 24x^2, -2x^{-\frac{1}{2}}, +3 - 2x^{-2}$ $\left[24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2} \right]$	M1 A1 M1 A1 A1A1
Notes		
<p>1st M1 for attempting to divide (one term correct)</p> <p>1st A1 for both terms correct on the same line, accept $3x^1$ for $3x$ or $\frac{2}{x}$ for $2x^{-1}$</p> <p>These first two marks may be implied by a correct differentiation at the end.</p> <p>2nd M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$ for at least one term of their expression</p> <p>“Differentiating” $\frac{3x^2 + 2}{x}$ and getting $\frac{6x}{1}$ is M0</p> <p>2nd A1 for $24x^2$ only</p> <p>3rd A1 for $-2x^{-\frac{1}{2}}$ allow $\frac{-2}{\sqrt{x}}$. Must be simplified to this, not e.g. $\frac{-4}{2}x^{-\frac{1}{2}}$</p> <p>4th A1 for $3 - 2x^{-2}$ allow $\frac{-2}{x^2}$. Both terms needed. Condone $3 + (-2)x^{-2}$</p> <p>If “+c” is included then they lose this final mark</p> <p>They do not need one line with all terms correct for full marks. Award marks when first seen in this question and apply ISW.</p> <p>Condone a mixed line of some differentiation and some division e.g. $24x^2 - 4x^{\frac{1}{2}} + 3x + 2x^{-1}$ can score 1st M1A1 and 2nd M1A1</p>		
Quotient /Product Rule	$\frac{x(6x) - (3x^2 + 2) \times 1}{x^2}$ or $6x(x^{-1}) + (3x^2 + 2)(-x^{-2})$ $\frac{3x^2 - 2}{x^2}$ or $3 - \frac{2}{x^2}$ (o.e.)	1 st M1 for an attempt: $\frac{P-Q}{x^2}$ or $R + (-S)$ with one of P, Q or R, S correct. 1 st A1 for a correct expression 4 th A1 same rules as above

Question Number	Scheme	Marks
<p>8.</p> <p>(a)</p>	$m_{AB} = \frac{4-0}{7-2} \left(= \frac{4}{5} \right)$ <p>Equation of AB is: $y-0 = \frac{4}{5}(x-2)$ or $y-4 = \frac{4}{5}(x-7)$ (o.e.)</p> $\underline{4x - 5y - 8 = 0} \text{ (o.e.)}$	<p>M1</p> <p>M1</p> <p>A1 (3)</p>
<p>(b)</p>	$(AB =) \sqrt{(7-2)^2 + (4-0)^2}$ $= \sqrt{41}$	<p>M1</p> <p>A1 (2)</p>
<p>(c)</p>	<p>Using isos triangle with $AB = AC$ then $t = 2 \times y_A = 2 \times 4 = 8$</p>	<p>B1 (1)</p>
<p>(d)</p>	<p>Area of triangle = $\frac{1}{2}t \times (7-2)$</p> $= \underline{20}$	<p>M1</p> <p>A1 (2)</p>
Notes		
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>DET</p>	<p>Apply the usual rules for quoting formulae here. For a correctly quoted formula with some correct substitution award M1 If no formula is quoted then a fully correct expression is needed for the M mark 1st M1 for attempt at gradient of AB. Some correct substitution in correct formula. 2nd M1 for an attempt at equation of AB. Follow through their gradient, not e.g. $-\frac{1}{m}$ Using $y = mx + c$ scores this mark when c is found. Use of $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$ scores 1st M1 for denominator, 2nd M1 for use of a correct point A1 requires integer form but allow $5y + 8 = 4x$ etc. Must have an “=” or A0</p> <p>M1 for an expression for AB or AB^2. Ignore what is “left” of the equals sign</p> <p>B1 for $t = 8$. May be implied by correct coordinates (2, 8) or the value appearing in (d)</p> <p>M1 for an expression for the area of the triangle, follow through their $t (\neq 0)$ but must have the (7-2) or 5 and the $\frac{1}{2}$.</p> <p>e.g. $\begin{matrix} 2 & 7 & 2 & 2 \\ 0 & 4 & t & 0 \end{matrix}$ Area = $\frac{1}{2}[8 + 7t + 0 - (0 + 8 + 2t)]$ Must have the $\frac{1}{2}$ for M1</p>	<p>8</p>

Question Number	Scheme	Marks
9.		
(a)	$a + 29d = 40.75$ or $a = 40.75 - 29d$ or $29d = 40.75 - a$	M1 A1 (2)
(b)	$(S_{30}) = \frac{30}{2}(a + l)$ or $\frac{30}{2}(a + 40.75)$ or $\frac{30}{2}(2a + (30 - 1)d)$ or $15(2a + 29d)$ So $1005 = 15[a + 40.75]$ *	M1 A1 cso (2)
(c)	$67 = a + 40.75$ so $a = (\pounds) 26.25$ or $2625p$ or $26\frac{1}{4}$ NOT $\frac{105}{4}$ $29d = 40.75 - 26.25$ $= 14.5$ so $d = (\pounds)0.50$ or 0.5 or $50p$ or $\frac{1}{2}$	M1 A1 M1 A1 (4)
	Notes	8
(a)	M1 for attempt to use $a + (n - 1)d$ with $n = 30$ to form an equation . So $a + (30 - 1)d =$ any number is OK A1 as written. Must see $29d$ not just $(30 - 1)d$. Ignore any floating £ signs e.g. $a + 29d = \pounds 40.75$ is OK for M1A1 These two marks must be scored in (a). Some may omit (a) but get correct equation in (c) [or (b)] but we do not give the marks retrospectively.	
	Parts (b) and (c) may run together	
(b)	M1 for an attempt to use an S_n formula with $n = 30$. Must see one of the printed forms. ($S_{30} =$ is not required) A1 cso for forming an equation with 1005 and S_n and simplifying to printed answer. Condone £ signs e.g. $15[a + \pounds 40.75] = 1005$ is OK for A1	
(c)	1 st M1 for an attempt to simplify the given linear equation for a . Correct processes. Must get to $ka = \dots$ or $k = a + m$ i.e. one step (division or subtraction) from $a = \dots$ Commonly: $15a = 1005 - 611.25 (= 393.75)$ 1 st A1 For $a = 26.25$ or $2625p$ or $26\frac{1}{4}$ NOT $\frac{105}{4}$ or any other fraction 2 nd M1 for correct attempt at a linear equation for d , follow through their a or equation in (a) Equation just has to be linear in d , they don't have to simplify to $d = \dots$ 2 nd A1 depends upon 2 nd M1 and use of correct a . Do not penalise a second time if there were minor arithmetic errors in finding a provided $a = 26.25$ (o.e.) is used. Do not accept other fractions other than $\frac{1}{2}$ If answer is in pence a "p" must be seen.	
Sim Equ	Use this scheme: 1st M1A1 for a and 2 nd M1A1 for d . Typically solving: $1005 = 30a + 435d$ and $40.75 = a + 29d$. If they find d first then follow through use of their d when finding a .	

Question Number	Scheme	Marks
<p>10. (a)</p> 	<p>(i) \cap shape (anywhere on diagram)</p> <p>Passing through or stopping at (0, 0) and (4,0) only (Needn't be \cap shape)</p> <p>(ii) correct shape (-ve cubic) with a max and min drawn anywhere</p> <p>Minimum or maximum at (0,0)</p> <p>Passes through or stops at (7,0) but <u>NOT</u> touching.</p> <p>(7, 0) should be to right of (4,0) or B0</p> <p>Condone (0,4) or (0, 7) marked correctly on x-axis. Don't penalise poor overlap near origin.</p> <p>Points must be marked on the sketch...not in the text</p> <p>(b)</p> $x(4-x) = x^2(7-x) \quad (0=)x[7x-x^2-(4-x)]$ $(0=)x[7x-x^2-(4-x)] \quad (\text{o.e.})$ $0 = x(x^2 - 8x + 4) \quad *$ <p>(c)</p> $(0 = x^2 - 8x + 4 \Rightarrow) x = \frac{8 \pm \sqrt{64-16}}{2} \quad \text{or} \quad (x \pm 4)^2 - 4^2 + 4 (= 0)$ $= \frac{8 \pm 4\sqrt{3}}{2} \quad \text{or} \quad (x-4)^2 = 12$ $x = 4 \pm 2\sqrt{3} \quad \text{or} \quad (x-4) = \pm 2\sqrt{3}$ <p>From sketch A is $x = 4 - 2\sqrt{3}$</p> <p>So $y = (4 - 2\sqrt{3})(4 - [4 - 2\sqrt{3}])$ (dependent on 1st M1)</p> $= -12 + 8\sqrt{3}$	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>(5)</p> <p>M1</p> <p>B1ft</p> <p>A1 cso (3)</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1 (7)</p> <p>15</p>
Notes		
<p>(b)</p> <p>(c)</p>	<p>M1 for forming a suitable equation</p> <p>B1 for a common factor of x taken out legitimately. Can treat this as an M mark. Can fit their cubic = 0 found from an attempt at solving their equations e.g. $x^3 - 8x^2 - 4x = x(\dots)$</p> <p>A1 cso no incorrect working seen. The “= 0” is required but condone missing from some lines of working. Cancelling the x scores B0A0.</p> <p>1st M1 for some use of the correct formula or attempt to complete the square</p> <p>1st A1 for a fully correct expression: condone + instead of \pm or for $(x-4)^2 = 12$</p> <p>B1 for simplifying $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{12} = 2\sqrt{3}$. Can be scored independently of this expression</p> <p>2nd A1 for correct solution of the form $p + q\sqrt{3}$: can be \pm or + or -</p> <p>2nd M1 for selecting their answer in the interval (0,4). If they have no value in (0,4) score M0</p> <p>3rd M1 for attempting $y = \dots$ using their x in correct equation. An expression needed for M1A0</p> <p>3rd A1 for correct answer. If 2 answers are given A0.</p>	

Question Number	Scheme	Marks
11.	<p>(a) $(y =) \frac{3x^2}{2} - \frac{5x^{\frac{1}{2}}}{\frac{1}{2}} - 2x (+c)$</p> <p>$f(4) = 5 \Rightarrow 5 = \frac{3}{2} \times 16 - 10 \times 2 - 8 + c$</p> <p style="text-align: right;"><u>$c = 9$</u></p> <p>$\left[f(x) = \frac{3}{2}x^2 - 10x^{\frac{1}{2}} - 2x + 9 \right]$</p> <p>(b) $m = 3 \times 4 - \frac{5}{2} - 2 \left(= 7.5 \text{ or } \frac{15}{2} \right)$</p> <p>Equation is: $y - 5 = \frac{15}{2}(x - 4)$</p> <p style="text-align: center;"><u>$2y - 15x + 50 = 0$</u> o.e.</p>	<p>M1A1A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1</p> <p>M1A1</p> <p>A1 (4)</p> <p>(9marks)</p>
Normal	<p>(a) 1st M1 for an attempt to integrate $x^n \rightarrow x^{n+1}$</p> <p>1st A1 for at least 2 correct terms in x (unsimplified)</p> <p>2nd A1 for all 3 terms in x correct (condone missing $+c$ at this point). Needn't be simplified</p> <p>2nd M1 for using the point (4, 5) to form a linear equation for c. Must use $x = 4$ and $y = 5$ and have no x term and the function must have "changed".</p> <p>3rd A1 for $c = 9$. The final expression is not required.</p> <p>(b) 1st M1 for an attempt to evaluate $f'(4)$. Some correct use of $x = 4$ in $f'(x)$ but condone slips. They must therefore have at least 3×4 or $-\frac{5}{2}$ and clearly be using $f'(x)$ with $x = 4$. Award this mark wherever it is seen.</p> <p>2nd M1 for using their value of m [or their $-\frac{1}{m}$] (provided it clearly comes from using $x = 4$ in $f'(x)$) to form an equation of the line through (4,5).</p> <p>Allow this mark for an attempt at a normal or tangent. Their m must be numerical. Use of $y = mx + c$ scores this mark when c is found.</p> <p>1st A1 for any correct expression for the equation of the line</p> <p>2nd A1 for any correct equation with integer coefficients. An "=" is required. e.g. $2y = 15x - 50$ etc as long as the equation is correct and has integer coefficients.</p> <p>Attempt at normal can score both M marks in (b) but A0A0</p>	